Modeling Layered Structures by Shell Elements Using the Finite Element Method

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A novel, two step method is presented to model thin or thick layered homogeneous or periodic structures replacing them by 2D sheets with shell elements. A unit cell model of the thin/thick 2D structure is used to extract the circuit *Y* matrix which characterizes the layer. Degenerated prism elements modeled as shell elements are applied by doubling the essential variables on the surface. The link between the two sides of the surface is governed by the circuit *Y* matrix obtained from the unit cell model. This way, shielding effects can be accurately and efficiently modeled by using the Finite Element Method without 3D meshing the layer(s).

***Index Terms*— Homogenization, Thin sheets, Finite Element Method (FEM), Shell elements, Periodic structures, Frequency Selective Surface (FSS).**

I. Introduction

IT is common when applying the Finite Element Method (FEM) to replace layers by sheet impedance boundary condition. A typical example for that is to model thin homogeneous or periodic layers of high conductivity and/or permeability. The main reasons for using the sheet impedance boundary condition is to better cope with the strong decay of the electromagnetic field in the layers and/or to avoid meshing of thin layered structures. There exist two typical cases where a simple sheet impedance boundary condition accurately replaces the layer(s): the electromagnetic field rapidly decays to zero inside the layer due to high conductivity or the field does not change much in the normal direction of the layer. The effective impedance boundary condition can be given by analytical formula or by numerically modeling a unit cell of the structure [1, 2]. When the skin depth of the layer is about the same or smaller than the thickness, the electromagnetic field will change considerably in the normal direction of the layer but not necessarily so rapidly that the field will be zero inside the layer. For this case, the surface currents of the two sides of the layer will not be the same as they begin to be decoupled which also results in the tangential components of the electric field on the two sides of the layer to be different. The simple sheet impedance boundary condition is no applicable for this case since it maintains continuous tangential electric field. Shell elements can be used for this case to provide accurate results.

Classical shell elements are degenerated prism elements created by doubling the tangential components of the electric field on the two sides of the layer [3]. Assuming constant, linear or higher order variation (shape functions) in the normal direction, the finite element matrix contributions are evaluated analytically in the shell by taking into account the fact that the layer is thin. The main goal is fulfilled by avoiding meshing the thin layer. There are limits to the applicability of classical shell elements especially when the layer is not thin enough such that the field varies significantly in the normal direction or the field decays strongly in the layer. In these cases, modified analytical models can help [4 - 6].

There are cases, when the structure cannot be characterized by closed analytical formulas. For example, periodic structures at high frequencies such as Frequency Selective Surfaces (FSSs) typically do not have known analytical formulas of surface impedance. The aim of this paper is to present a two stage method which does not need analytical coupling formulas to accurately replace FSSs by a sheet with shell elements. The first step is to extract a ***Y*** matrix of the FSS by solving a unit cell model of the layer similarly to what was proposed in [3]. The second step is to use the ***Y*** matrix to couple the two sides of the layer in the FEM. This method enables shielding effects to be accurately and efficiently modeled for an arbitrary, thin or even thick FSS. The proposed shell elements should be valid for any frequency and material composition. Asymptotic behaviors and singularity treatments will also be presented in the final version in order to have a very robust broad frequency band shell element implementation.

To validate the special shell element method, shielding of a homogeneous copper layer was investigated. Shielding effectiveness was calculated by shell elements and compared to the reference solution got by using 3D FEM with very fine mesh. Results from real life practical FSS structures will be presented in the final version.

II. Mathematical Model

The goal is to set up an internal sheet boundary condition, which is equivalent to a layer (thin or thick), which can be seen in Fig. 1.

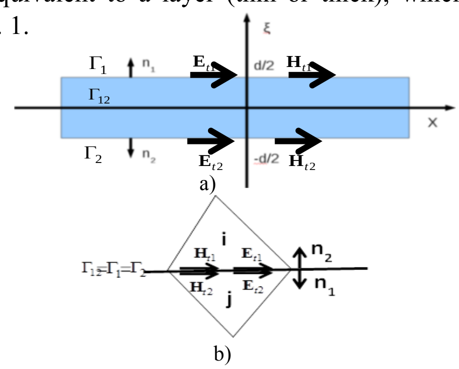

Fig. 1. a) A layer to be replaced by an equivalent sheet boundary condition b) Two adjacent finite element tetrahedra on Γ_{12}.

The field relationship between the two sides of the layer can be written as:

$$\begin{bmatrix} \mathbf{n}\times\mathbf{H}_{t1} \\ \mathbf{n}\times\mathbf{H}_{t2} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \mathbf{n}\times\mathbf{n}\times\mathbf{E}_{t1} \\ \mathbf{n}\times\mathbf{n}\times\mathbf{E}_{t2} \end{bmatrix} \quad (1)$$

where the frequency dependent ***Y*** matrix characterizes the layer(s). It can be given in closed analytical form, e.g. by using a transmission line model of the layer ([4 - 6]) or in numerical form by linking to a FEM unit cell model of the layer [2]. The latter is the aim of this paper.

Since the relationship is known between the two sides of the layer, the 3D layer can be excluded from the computational domain by shrinking the thickness to zero and doubling the essential variables on Γ_{12}. This results in two regular Neumann boundary conditions on the two sides of Γ_{12}:

$$\int_{\Gamma_1} \mathbf{E}_1^* \bullet (\mathbf{n}_1 \times \nu_r curl \mathbf{E}_1) d\Gamma =$$
$$- j\omega\mu_o \int_{\Gamma_1} (\mathbf{E}_1^* \bullet (\mathbf{n}_1 \times (y_{11}\mathbf{n}_1 \times \mathbf{E}_1) + (y_{12}\mathbf{n}_1 \times \mathbf{E}_2)) d\Gamma \quad (2)$$

$$\int_{\Gamma_2} \mathbf{E}_2^* \bullet (\mathbf{n}_2 \times \nu_r curl \mathbf{E}_2) d\Gamma =$$
$$- j\omega\mu_o \int_{\Gamma_2} (\mathbf{E}_2^* \bullet (\mathbf{n}_1 \times (y_{22}\mathbf{n}_2 \times \mathbf{E}_2) + (y_{21}\mathbf{n}_2 \times \mathbf{E}_1)) d\Gamma \quad (3)$$

where $\boldsymbol{E}^*$ denotes weighting functions in the FEM Galerkin method.

This way, a shell element is similar to an outer impedance boundary condition.The coupling between the two sides is implemented via the terms with y_{12} and y_{21} by doubling degrees of freedoms of ***E*** on the surface. It also follows, when assembling the *i*-th tetrahedron, no coupling terms are including from the *j*-th tetrahedron except the y_{12} term and similarly for the *j*-th tetrahedron.

Note that the method is also valid for thick layers. In that case, the thickness cannot be removed but the interior does not need to be meshed. The surface mesh of the two sides does not need to be conformal. The method is applicable both for layers with high conductivity and dielectric layers. For dielectric layers, the accuracy depends on the incident angle of the field. This dependence is much less when using layers with high conductivity. The derivations above considered a thin large layer "floating" in other materials. The final version will discuss the case when the layers touch each other. Special handling is needed on the border of shell sheets where an element has both split edges and non-split edges [7]. In addtion, the ***Y*** matrix can become singular which results in a singular system matrix. These complications will also be discussed in the final version.

III. Verification

The shielding efficiency of a 4 um metal layer ($\sigma = 5.0e7$S/m) has been studied both by a very fine discretized 3D FEM solution as reference and by shell elements presented in the paper. The unit cell model of the layer can be seen in Fig.2. A parellel plate waveguide was used with two 2um layers to obtain good accuracy for the ***Y*** matrix. The ***Y*** matrix was applied to the test case of a large sheet of shell element boundary condition in the middle of parallel plate waveguide (see Fig. 3). Fig. 4 shows the peak electric field along the dashed central line of Fig. 3. The results of shell elements by using the ***Y*** matrix (red curves) extracted from the unit cell model agree well with the reference (green curves). The curves are on top of each other at low freuqencies. The unit cell can be the model of any complicated FSS.

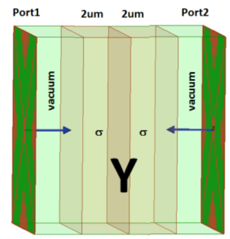

Fig. 2. Unit cell of a 4um thick metal layer to calculate **Y** matrix

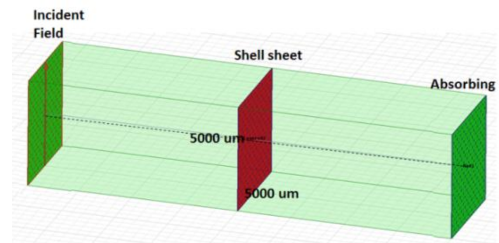

Fig. 3. Shielding of a sheet in parallel plate waveguide

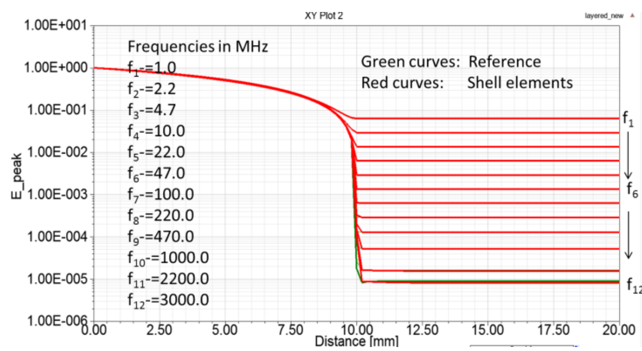

Fig. 4. Shielding efficiency (**E** field normalized to 1) along the dashed central line of Fig.3.

IV. References

[1] I. Bardi, M. Vogel and Z. Cendes, "Modeling Large Screens via Homogenization with the Finite Element Method," *Microwave Symposium Digest, IEEE MTT-S International*, pp. 1315 - 1318, 2006.

[2] I. Bardi, J. Tharp and R. Petersson, "Homogenization of Periodic Structures Using the FEM," *IEEE Trans on Magnetics*, Vol 50, No *2***,** pp. 157-160, February, 2014.

[3] Z. Ren, "Degenerated Whitney Prism Elements - General Nodal and Edge Shell Elements for Field Computation in Thin Structures, " *IEEE Transactions on Magnetics*, Vol. 34, No. *5,* September 1998.

[4] T. Abboud, F. Béreux, "Thin Conducting Sheet (TCS) in non-destructive testing simulations: implementation in Code_Carmel3D and validation," *Compumag* Budapest, PD2-16, Budapest, Hungary, 2013.

[5] O. Biro, I. Bardi, K. Preis, W. Renhart and K. Richter,"A Finite Element Formulation for Eddy Current Carrying Ferromagnetic Thin Sheets , " *IEEE Transactions on Magnetics*, Vol 33, No *2***,** March 1997.

[6] W. Renhart, C. Magele, C. Tuerk, "Thin Layer Transition Matrix description applied to the Finite Element Method," *IEEE Transactions on Magnetics*, Vol 45, No *5***,** pp. 1618-1621,March 2009..

[7] R. A. Ehlers and A. C. Ricky Metaxas, "3-D FE Discontinuous Sheet for microwave Heating", *IEEE Trans. MTT*, Vol. 51, No. 3, March 2003.